PG 2024
Huangshan, China

A Moving Least-Squares/Level-Set Particle Method for Bubble and Foam Simulation

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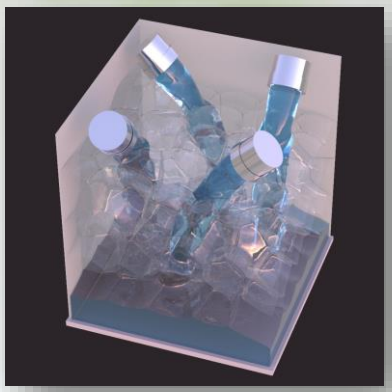
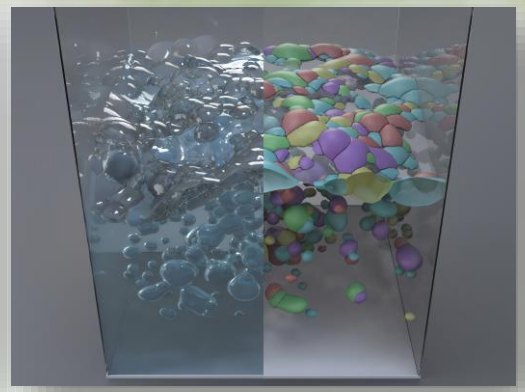
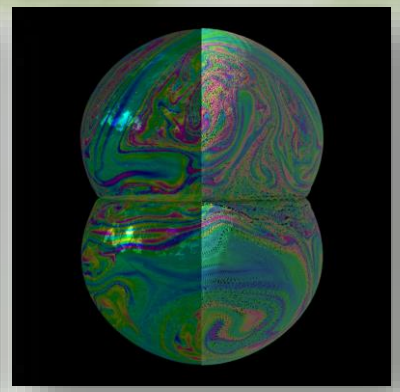
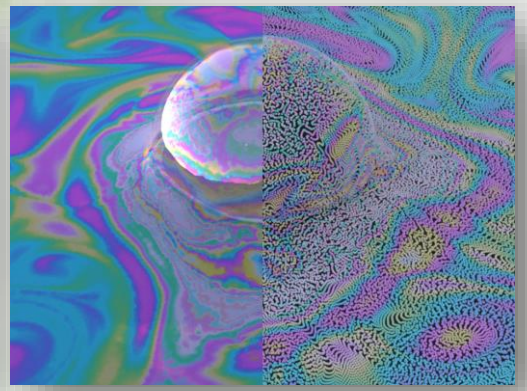
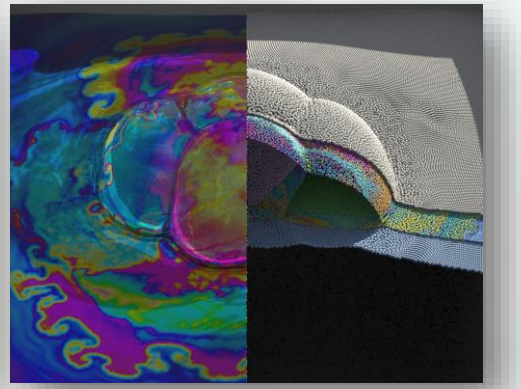
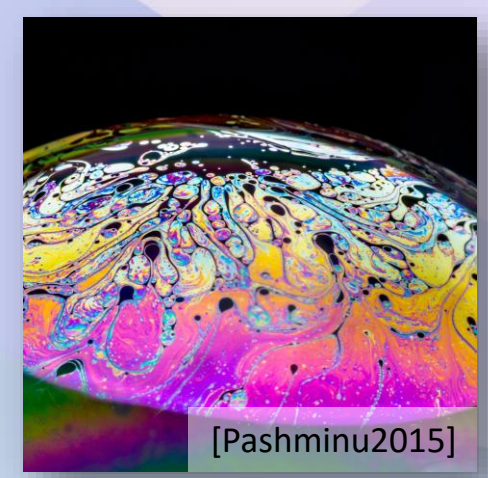
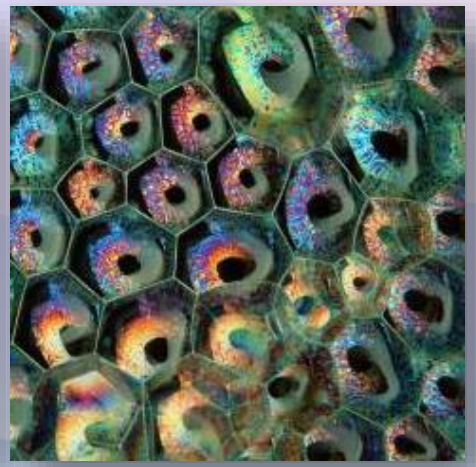






Motivation

Bubbles

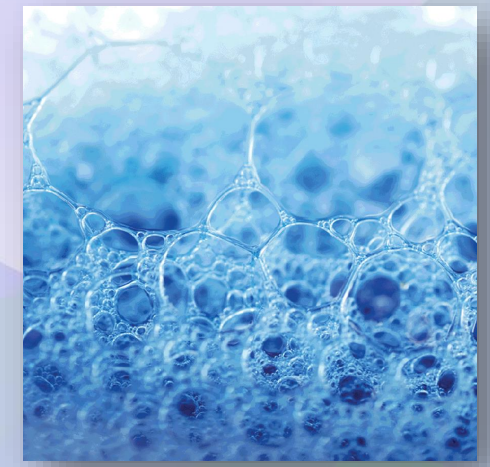
- Thin films with varying thickness and surfactant
- Intricate interfacial flow on non-manifold surfaces

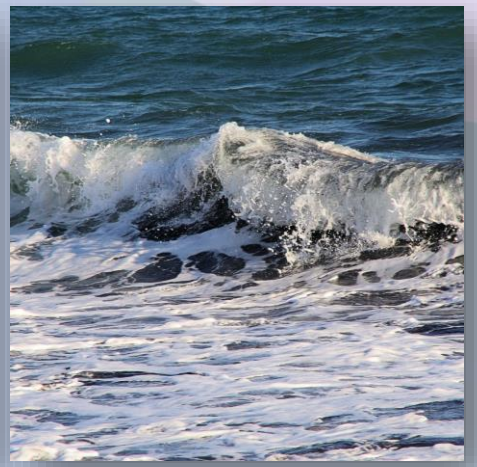




Foams

- Multiphase multi-regional fluid system
- Highly dynamic interactions between regions

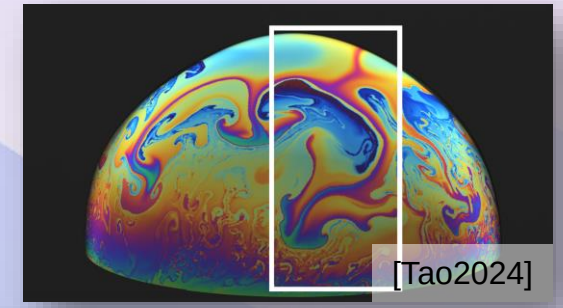


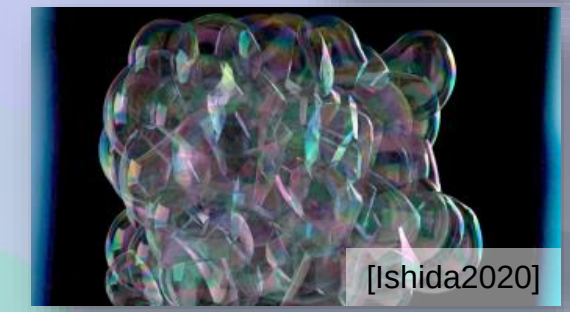


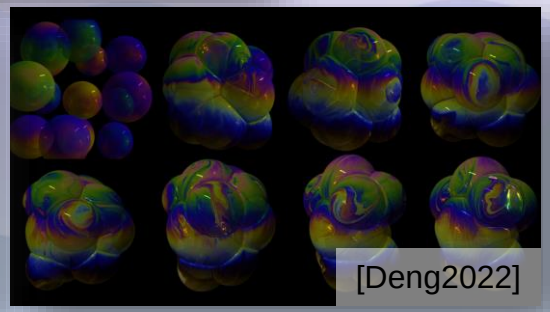
Literature Review

Bubbles

- Mesh / Codimensional Particles


[Tao2024]


[Ishida2020]


[Deng2022]

Foams

- Grid / Particles


[Patkar2013]

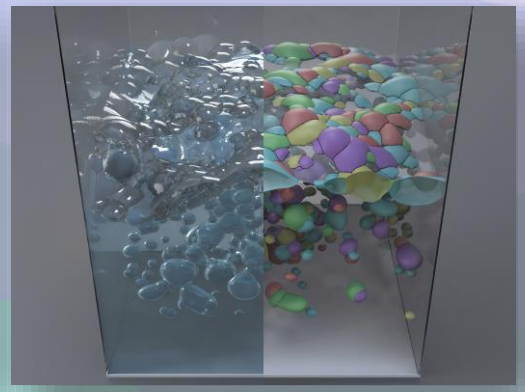
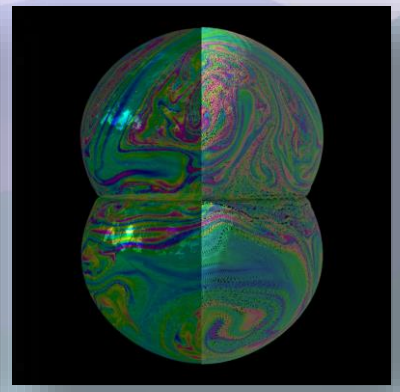
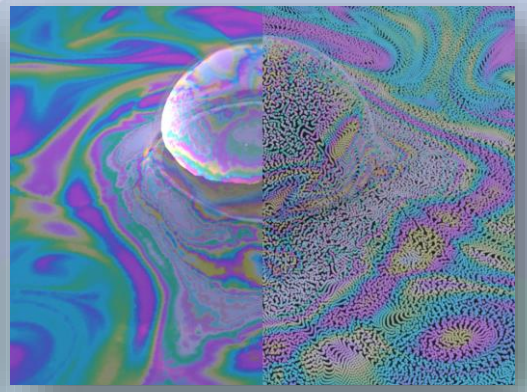
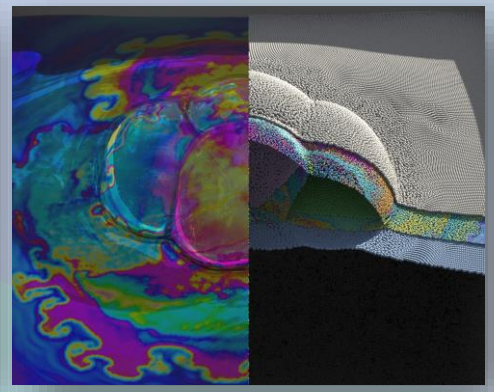

[Busaryev2012]


[Qu2023]


[Karnakov2022]

Main Contributions

- A novel particle-grid representation for multiphase interface tracking
  - Moving Least Squares + Particle level set
- A coupled system for multiphase bubble/foam physics
  - Multiphase foaming flow + Thin-film interfacial flow
- A unified simulation framework
  - Various bubble and foam phenomena

Geometric Representation

Particle-grid hybrid representation and their collaboration

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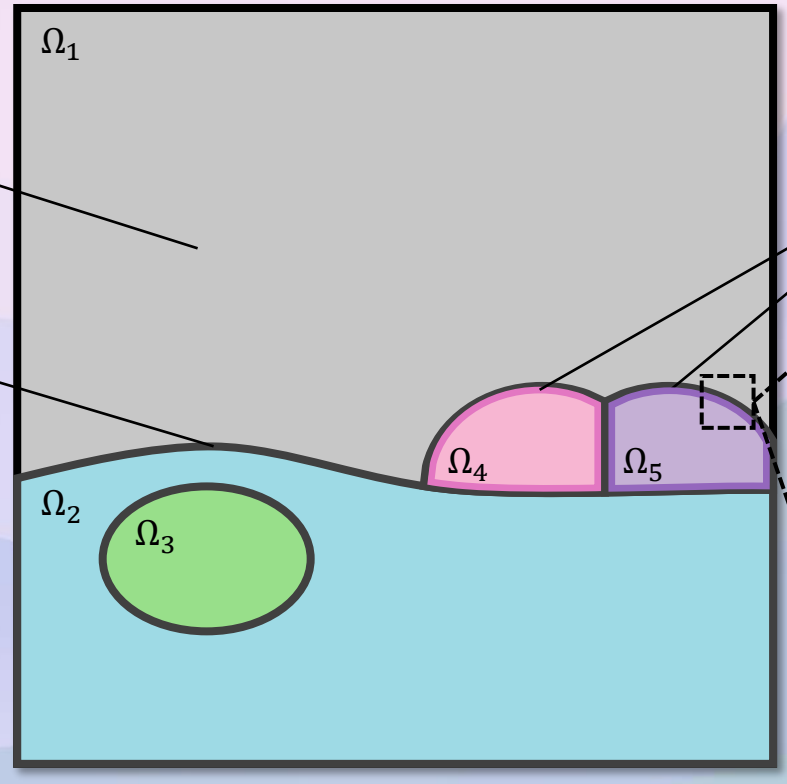
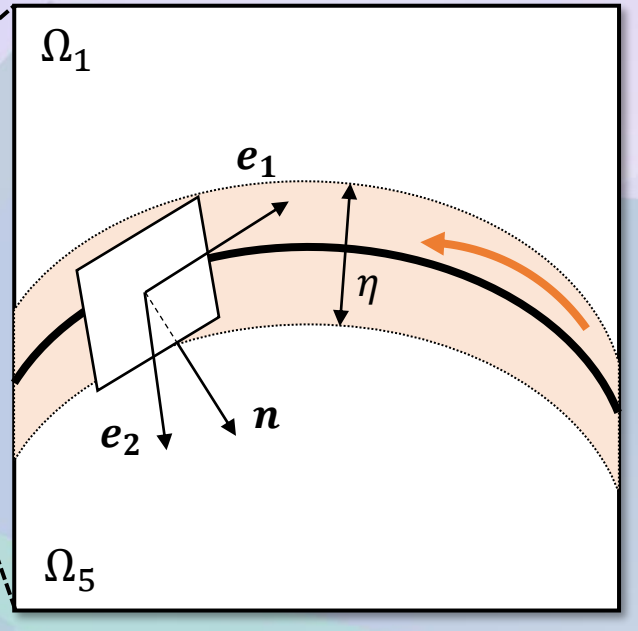
Domain Definition

Volumetric Regions Ω_i

Interfaces $\cup_i (\partial\Omega_i)$

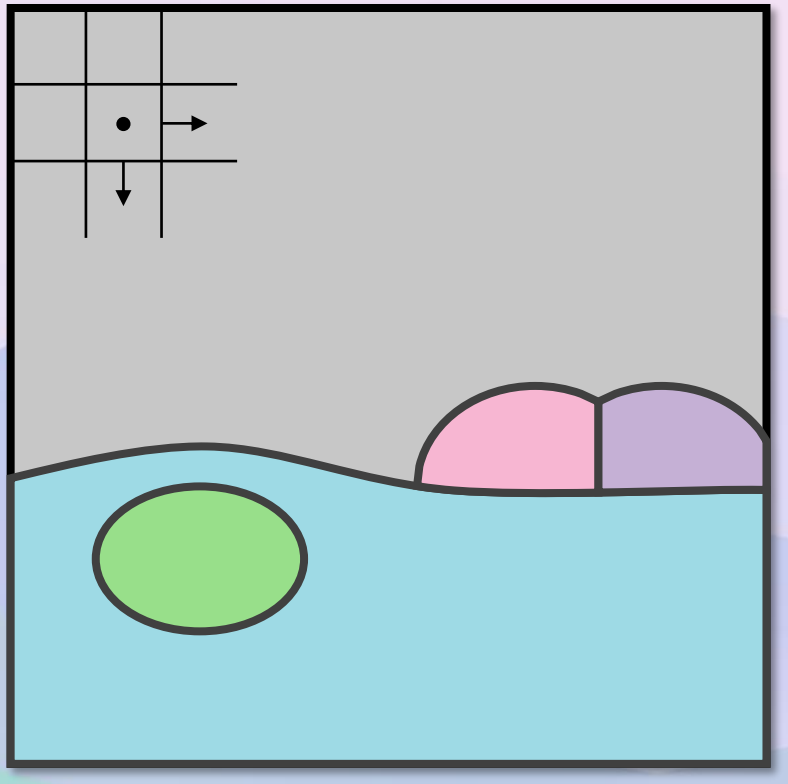


Region Interfaces $\partial\Omega_i$

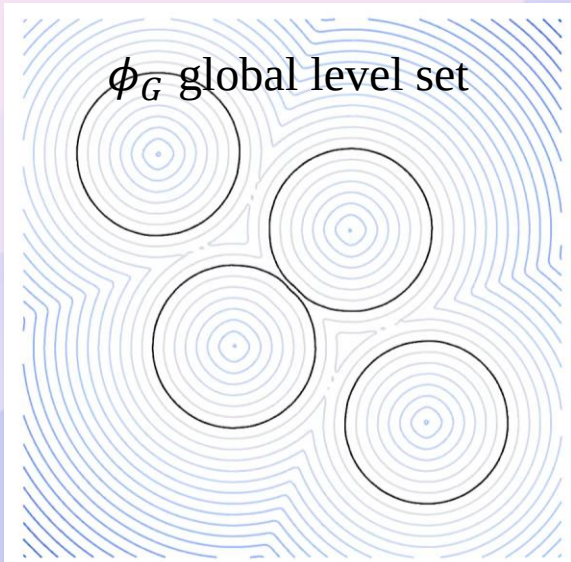


Interface Discretization

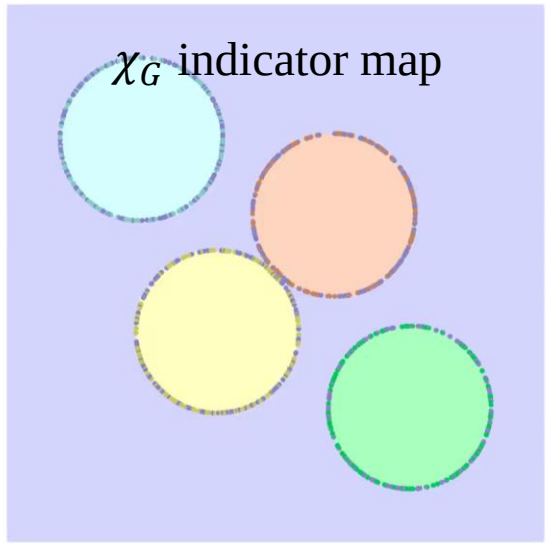
Grid $\mathcal{G}$



MLSLS Particle $\mathcal{E}$



Lagrangian Particle $\mathcal{L}$



↦ $\mathbf{u}_G$ velocity field

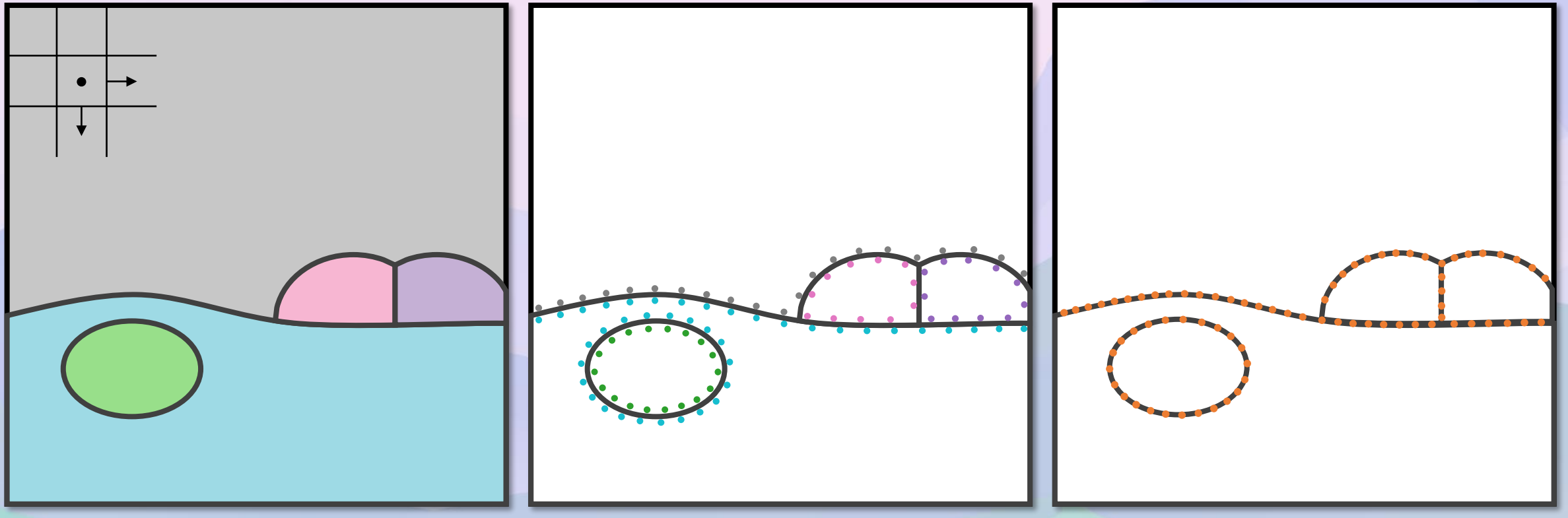
⊡ ϕ_G global level set

χ_G indicator map ( )

Interface Discretization

(Moving Least-Squares/Level-Set)

Grid $\mathcal{G}$ | MLSLS Particle $\mathcal{E}$ | Lagrangian Particle $\mathcal{L}$



$\mapsto$ $\mathbf{u}_G$ velocity field

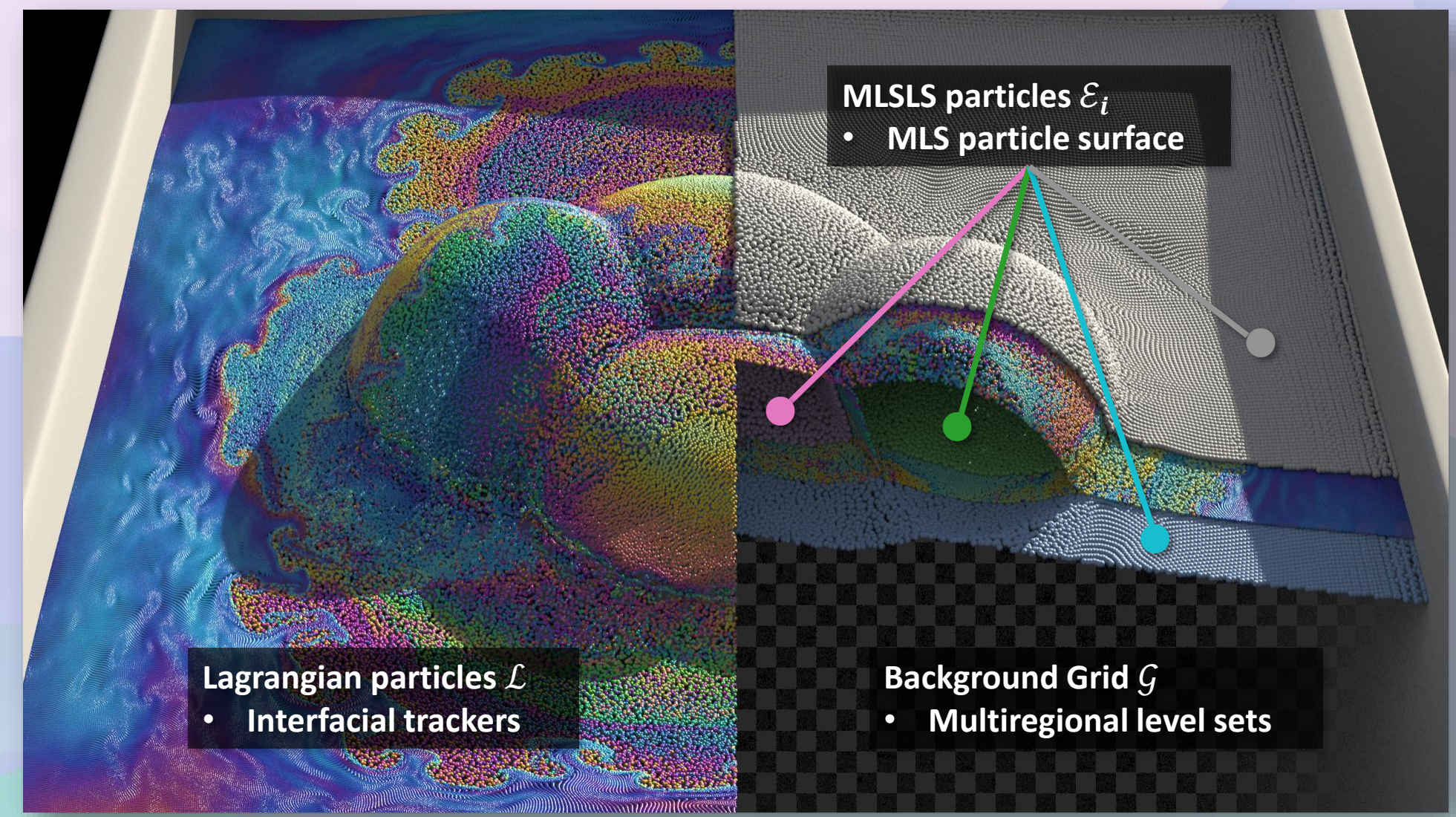
$\odot$ ϕ_G global level set

χ_G indicator map ( )

$\mathcal{E}_i$ ( ) MLSLS particles on $\partial\Omega_i$

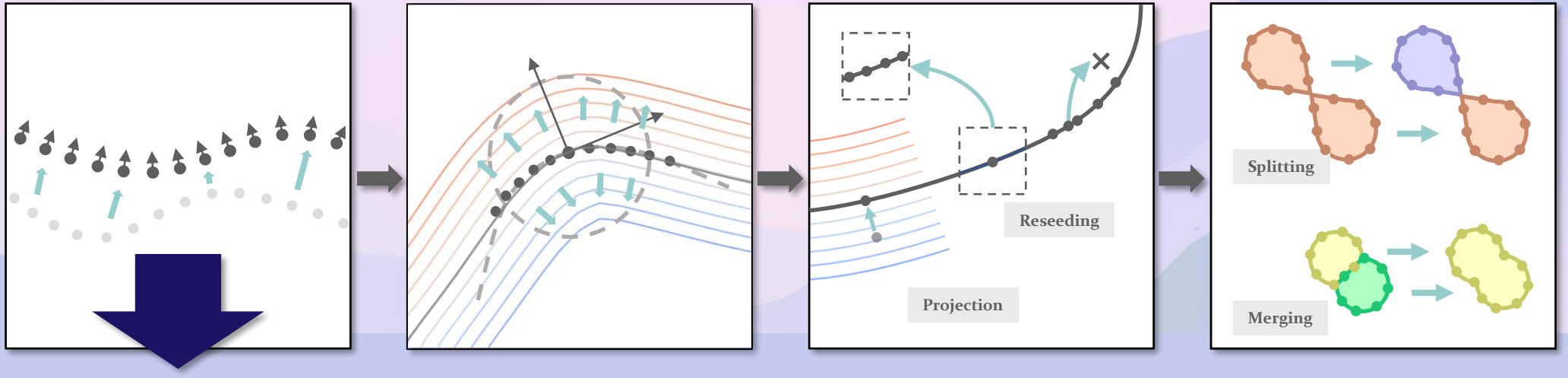
$\mathcal{L}$ Lagrangian particles on $\cup_i (\partial\Omega_i)$

Interface Discretization



Geometric Evolution

1 MLSLS Advection | 2 P2G Propagation | 3 G2P Correction | 4 Topo Evolution

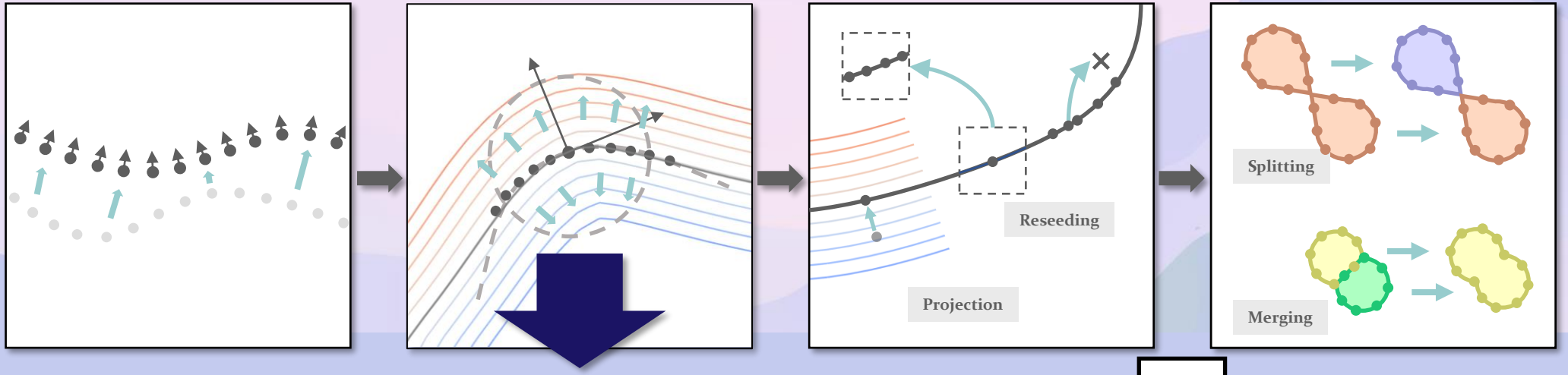


1 MLSLS Particle Advection

$$x_p^{new} \leftarrow x_p + u(x_p)\Delta t$$
$$n_p^{new} \leftarrow n_p - \nabla u^T(x_p) n_p \Delta t \quad \text{,for } p \in \forall \mathcal{E}_i$$

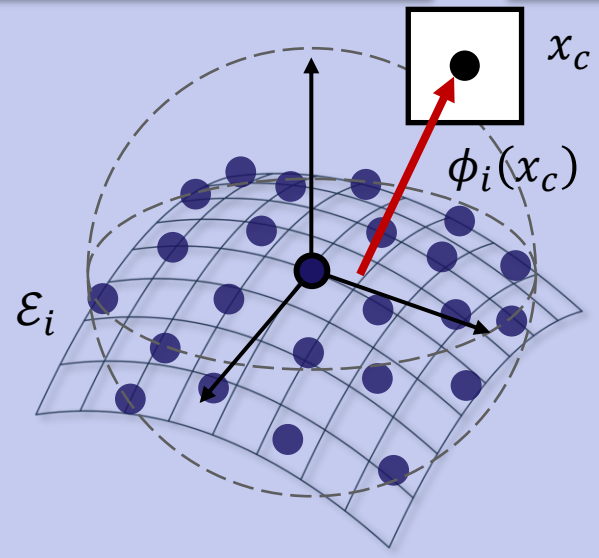
Geometric Evolution

1 MLSLS Advection | 2 P2G Propagation | 3 G2P Correction | 4 Topo Evolution



2 Regional Level Set Reconstruction

- Fit local MLS surface with particles $\mathcal{E}_i$
- Approximate regional level set ϕ_i



Geometric Evolution

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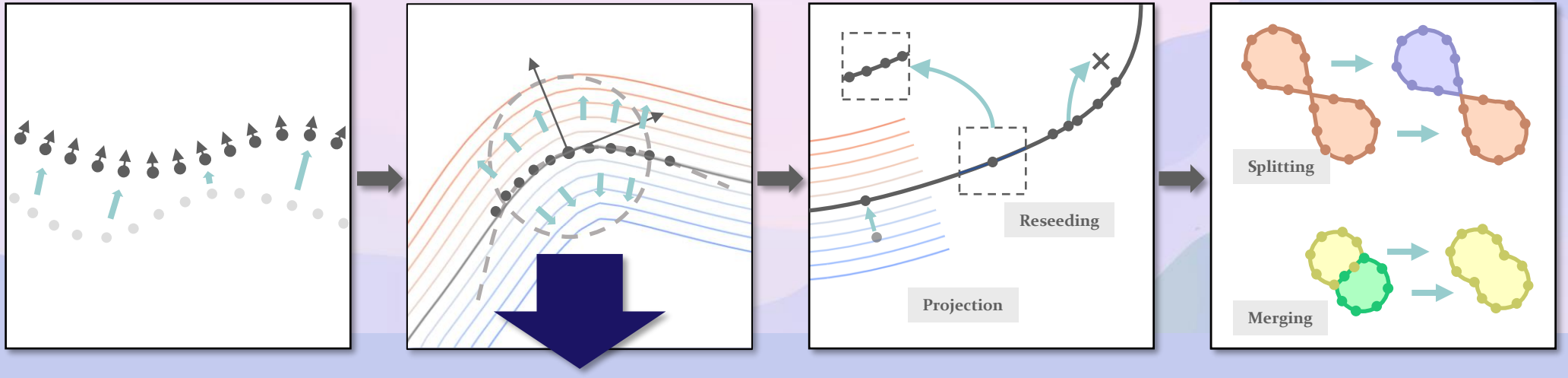
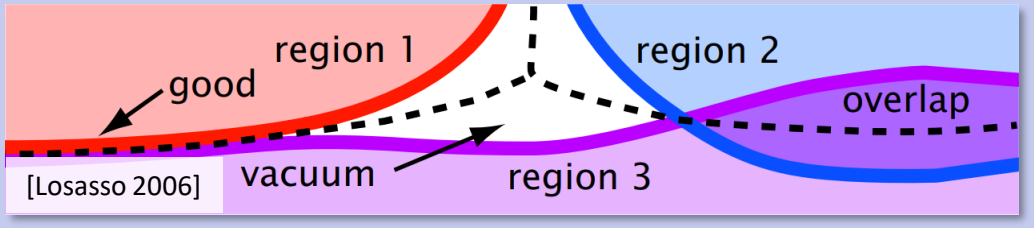


2 Regional Level Set Reconstruction

- Fit local MLS surface with particles $\mathcal{E}_i$
- Approximate regional level set ϕ_i

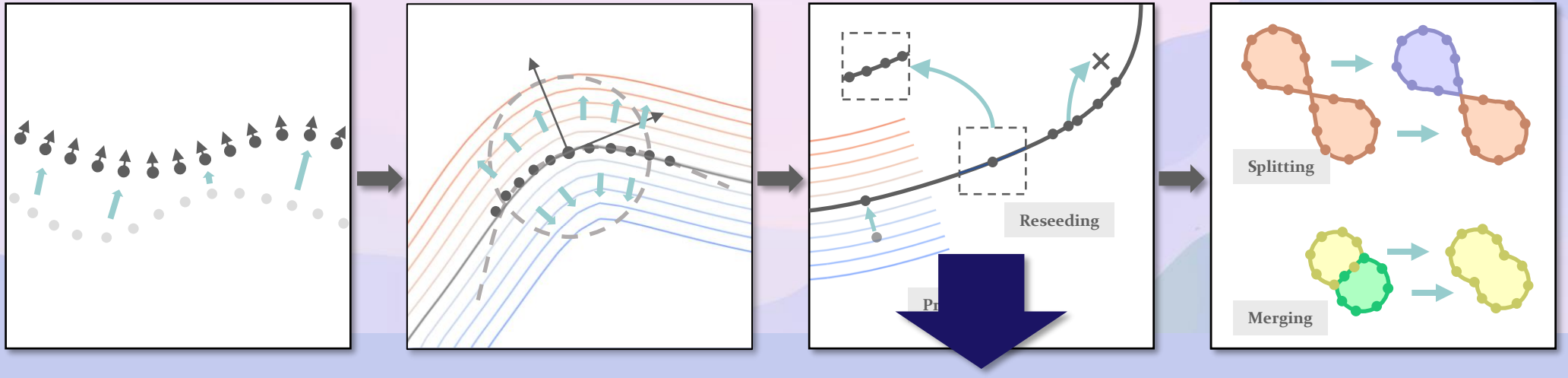
Global Level Set Reconstruction

- Fix vacuum or overlap between regions
- $\phi_G(x) = \min_i(\phi_i(x))$ and $\chi_G(x) = \arg\min_i(\phi_i(x))$



Geometric Evolution

1 MLSLS Advection | 2 P2G Propagation | 3 G2P Correction | 4 Topo Evolution



3 Particle Projection

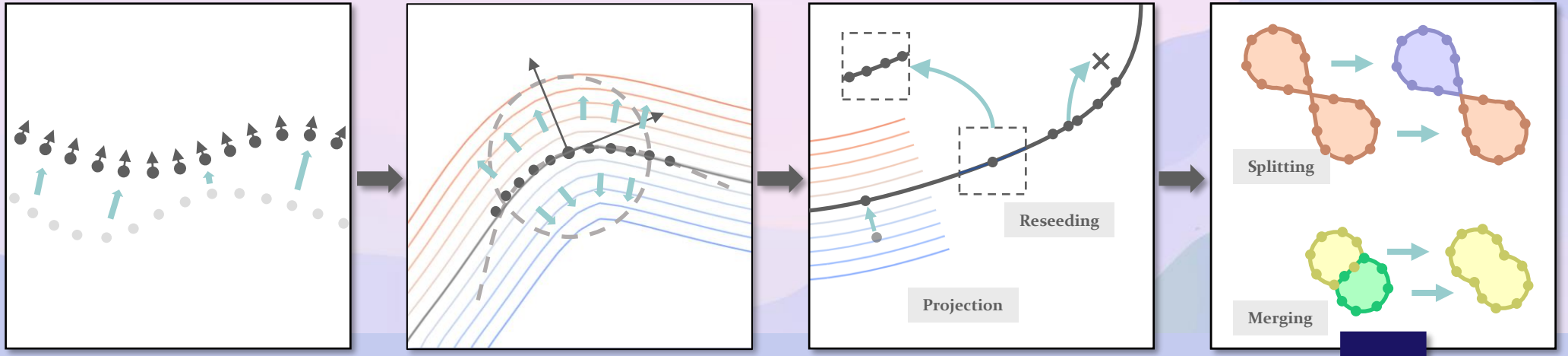
- Project MLSLS particles $\mathcal{E}_i$ onto $\phi_i = 0$

Particle Reseeding

- Insert particles in FLIP-like manner
- Remove densely clustered particles

Geometric Evolution

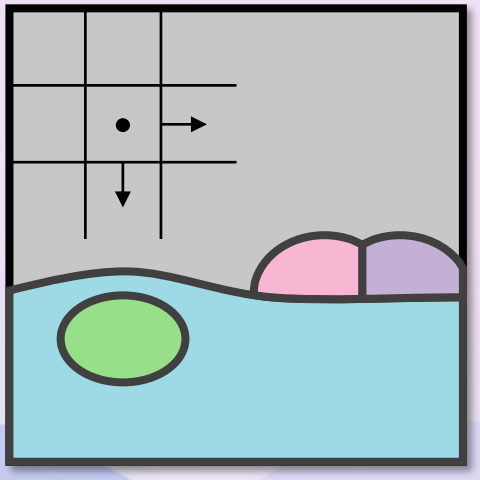
1 MLSLS Advection | 2 P2G Propagation | 3 G2P Correction | 4 Topo Evolution



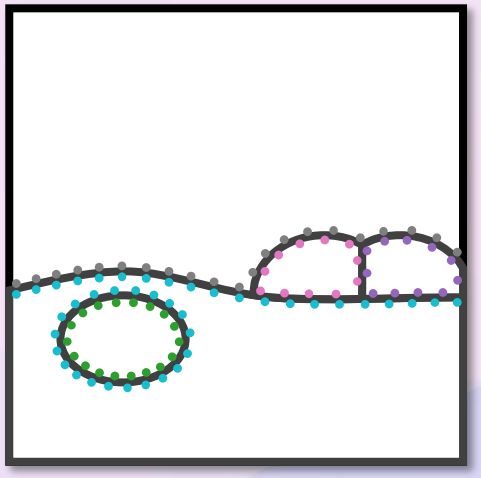
4 Splitting & Merging

- Handle topological change on the global level set ϕ_G and indicator map χ_G
- Split/Merge MLSLS particle set $\mathcal{E}_i$

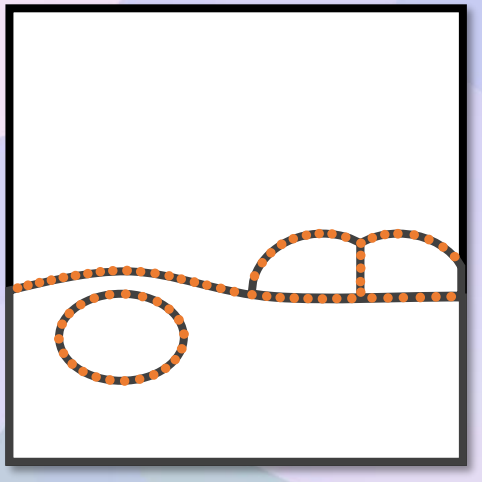
Geometric Evolution



Grid $\mathcal{G}$



MLSLS Particle $\mathcal{E}$



Lagrangian Particle $\mathcal{L}$

Co-evolve + Co-calibrate

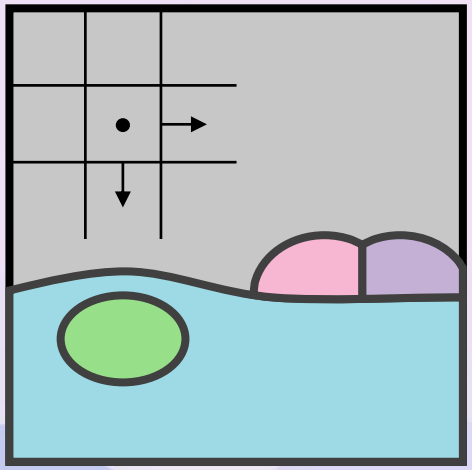
Move upon

Interface Geometry

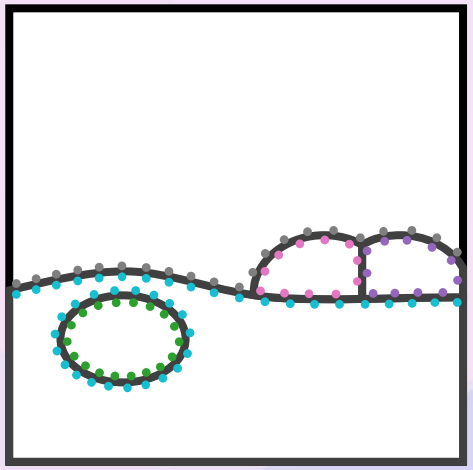
Simulation Framework

Volumetric multiphase flow and interfacial flow

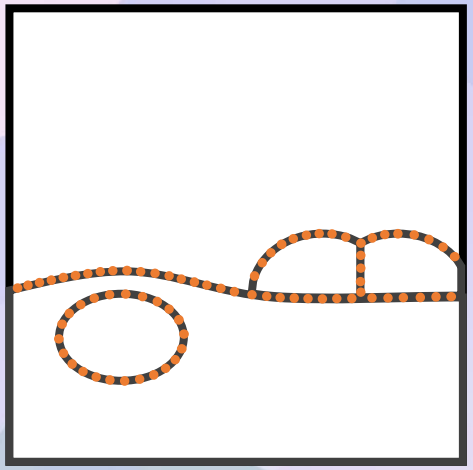
Simulation Framework



Grid $\mathcal{G}$



MLSLS Particle $\mathcal{E}$

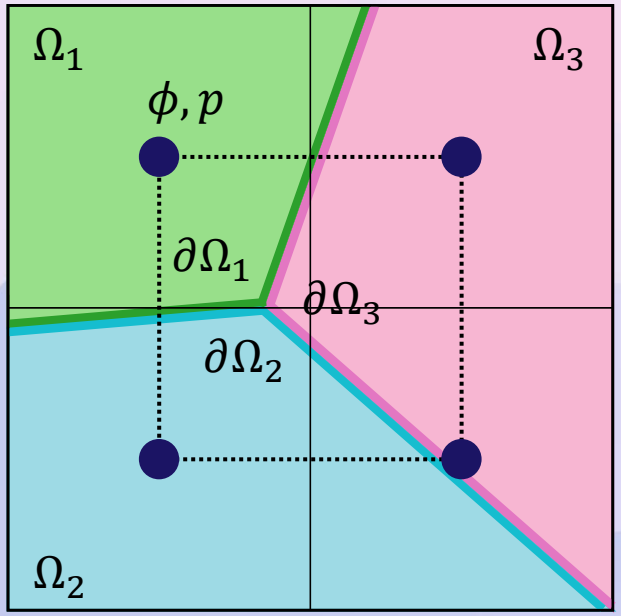


Lagrangian Particle $\mathcal{L}$

Volumetric Multiphase Flow

Interfacial Flow

Volumetric Multiphase Flow



Physics model

$$\begin{cases} \dfrac{\partial u}{\partial t} + u \cdot \nabla u = -\dfrac{\nabla p}{\rho_i} + g \\ \nabla \cdot u = 0 \end{cases}, x \in \Omega$$

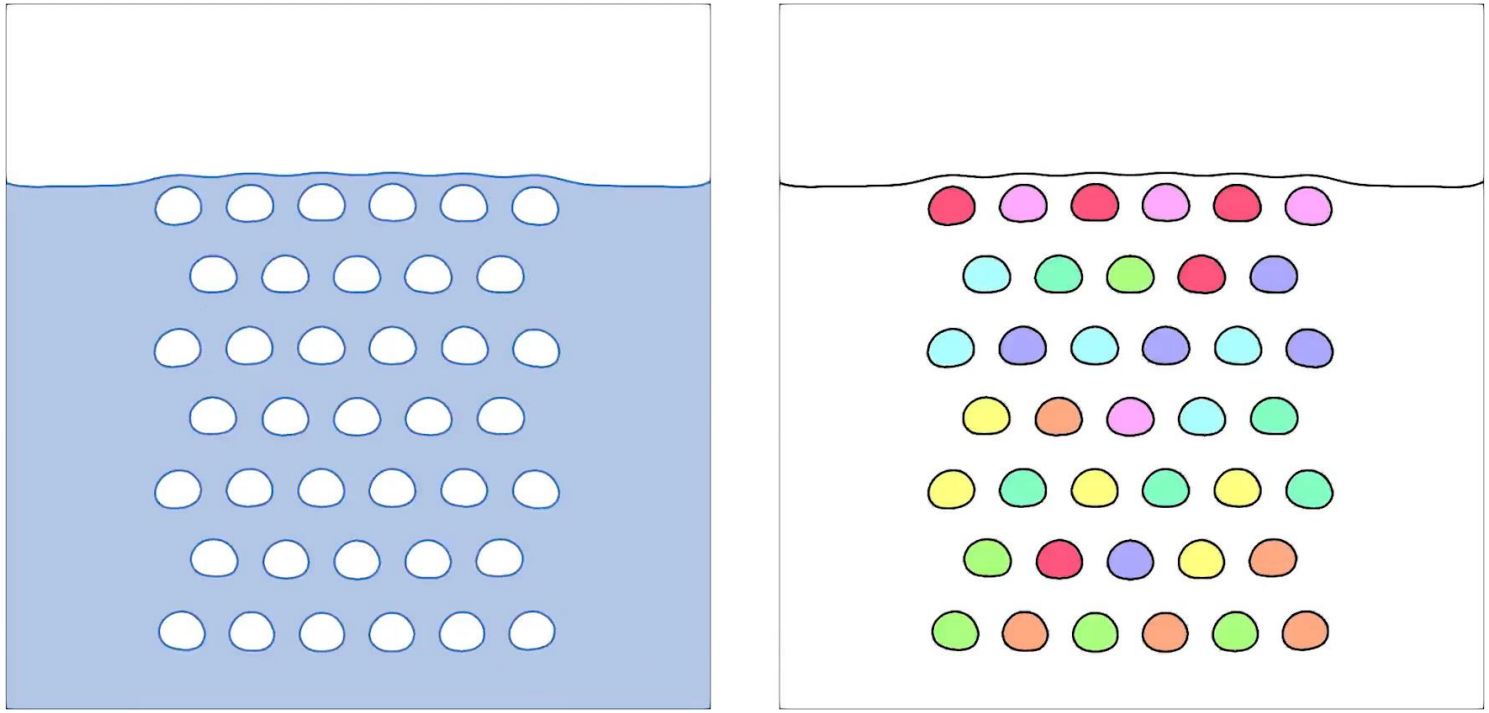
$$\begin{cases} [p] = c\gamma\kappa \\ [u] = 0 \end{cases}, x \in \partial\Omega$$

Dynamics Computation

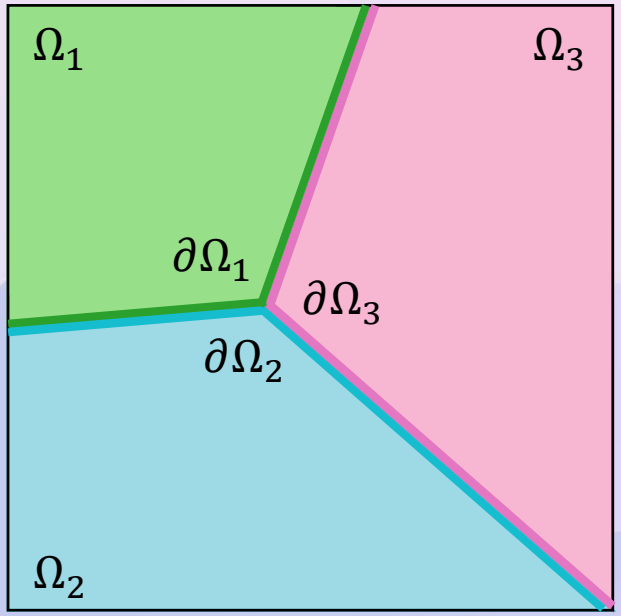
$$\begin{cases} u_G^* = u_G - \dfrac{\Delta t}{\rho}\nabla p + \delta_s(x)\dfrac{\Delta t}{\rho}\gamma\kappa n \\ \nabla \cdot u_G^* = 0 \end{cases}$$

Volumetric Multiphase Flow

2D Rising Bubbles



Interfacial Flow



Physics model:

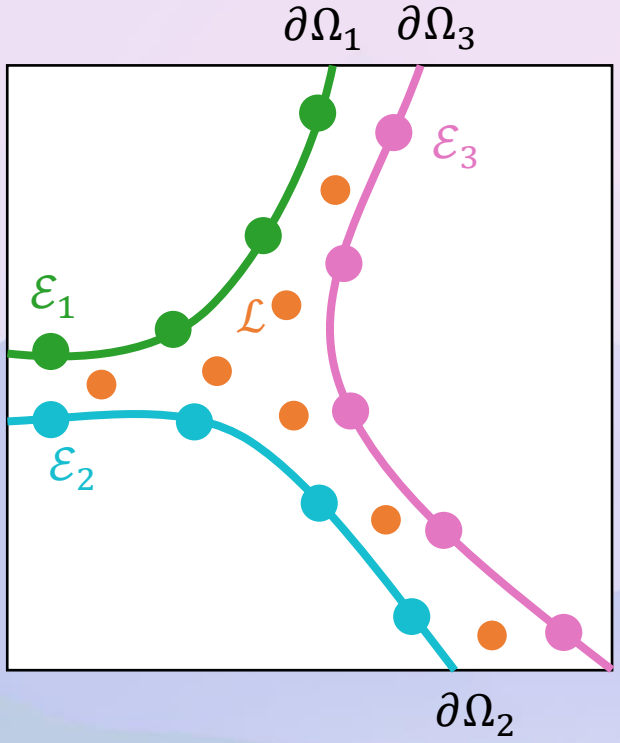
Independent tangential material transport [Deng2022]

$$\begin{cases} \dfrac{Du^{\top}}{Dt} = -\dfrac{2\bar{R}T}{\rho\eta}\nabla_s\Gamma + \dfrac{1}{\rho}g^{\top} \\ \dfrac{D\Gamma}{Dt} = -\Gamma\nabla_s \cdot u \\ \dfrac{D\eta}{Dt} = -\eta\nabla_s \cdot u \end{cases} , x \in \partial\Omega$$

- $u^{\top}$ Tangential velocity
- Γ Surfactant concentration
- η Thin film thickness

- $\bar{R}$ Gas constant
- T Temperature

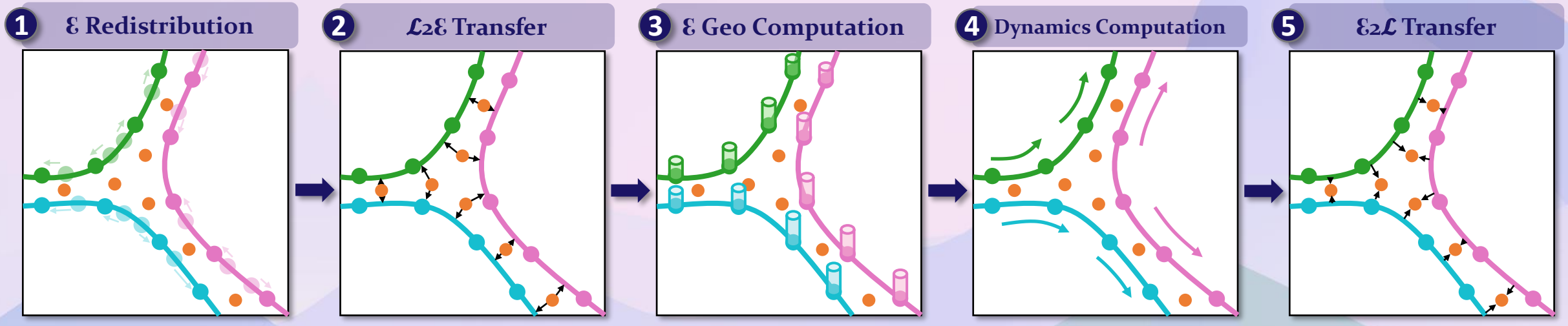
Interfacial Flow



Discretization

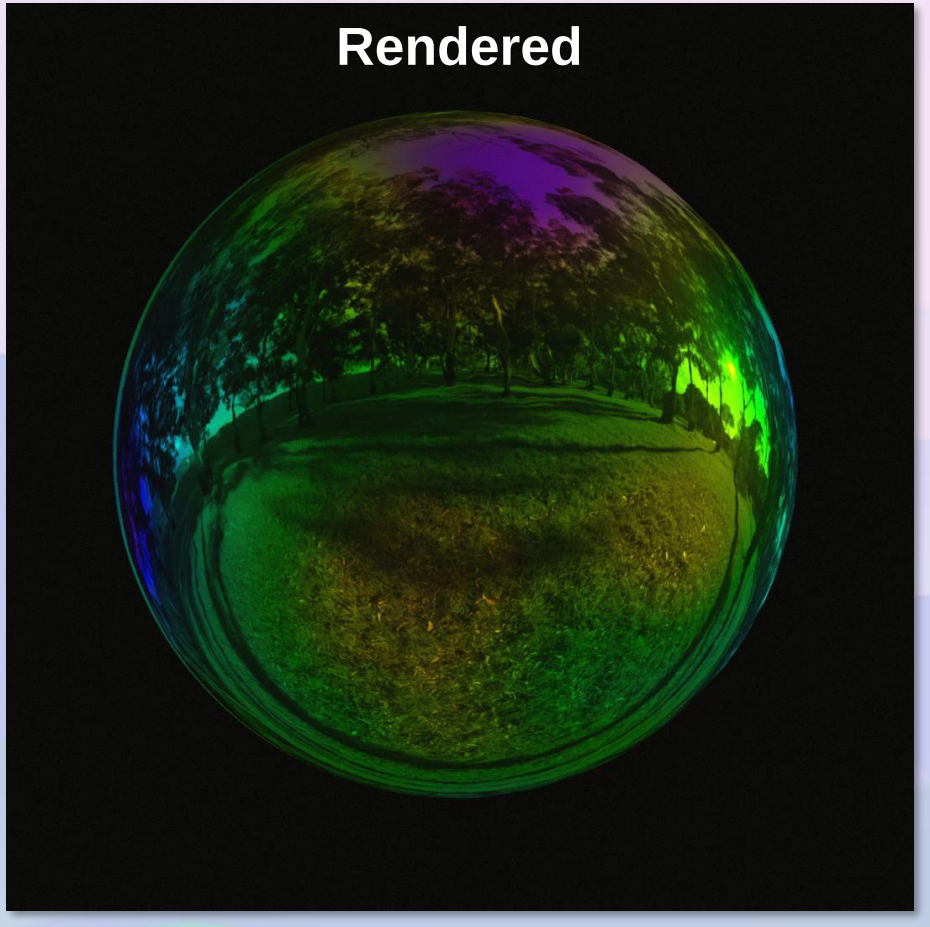
- MLSLS particle $\mathcal{E}_i$ as surface discretization
- Lagrangian particle $\mathcal{L}$ as material trackers
  - Mass m_L
  - Surfactant c_L
  - Volume V_L
  - Tangential momentum $p_L^{\top}$
  - Thickness η_L

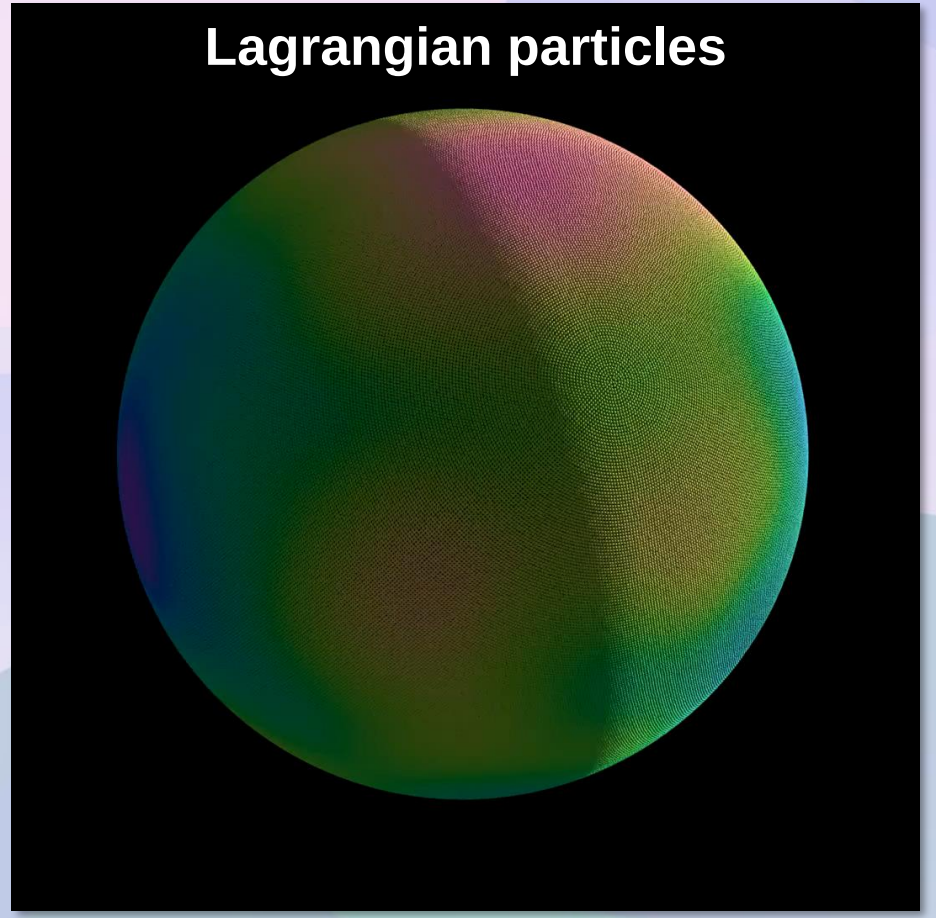
Interfacial Flow



1. Solve pseudo pressure on $\mathcal{E}_i$ to ensure a uniform distribution
2. Transfer physical quantities from $\mathcal{L}$ to $\mathcal{E}_i$ of their two closest regions
3. Compute geometry related properties (a_E, η_E, Γ_E); Fit MLS surface on $\mathcal{E}_i$ to build differential operators (∇_s, ∇_s^2)
4. Solve interfacial flow equation to obtain $\boldsymbol{u}_E^{\mathsf{T}}$
5. Transfer updated tangential velocity backward from $\mathcal{E}_i$ to $\mathcal{L}$

Interfacial Flow





Simulation Pipeline

1. MLSLS Particle Advection
2. P2G Propagation
3. G2P Correction
4. Topo Evolution

➢ **Geometry Evolution**

5. Velocity Advection

6. Volumetric Multiphase Flow Solving
7. Interfacial Flow Solving

➢ **Dynamics Solving**

Results

Bubbles, foams and more

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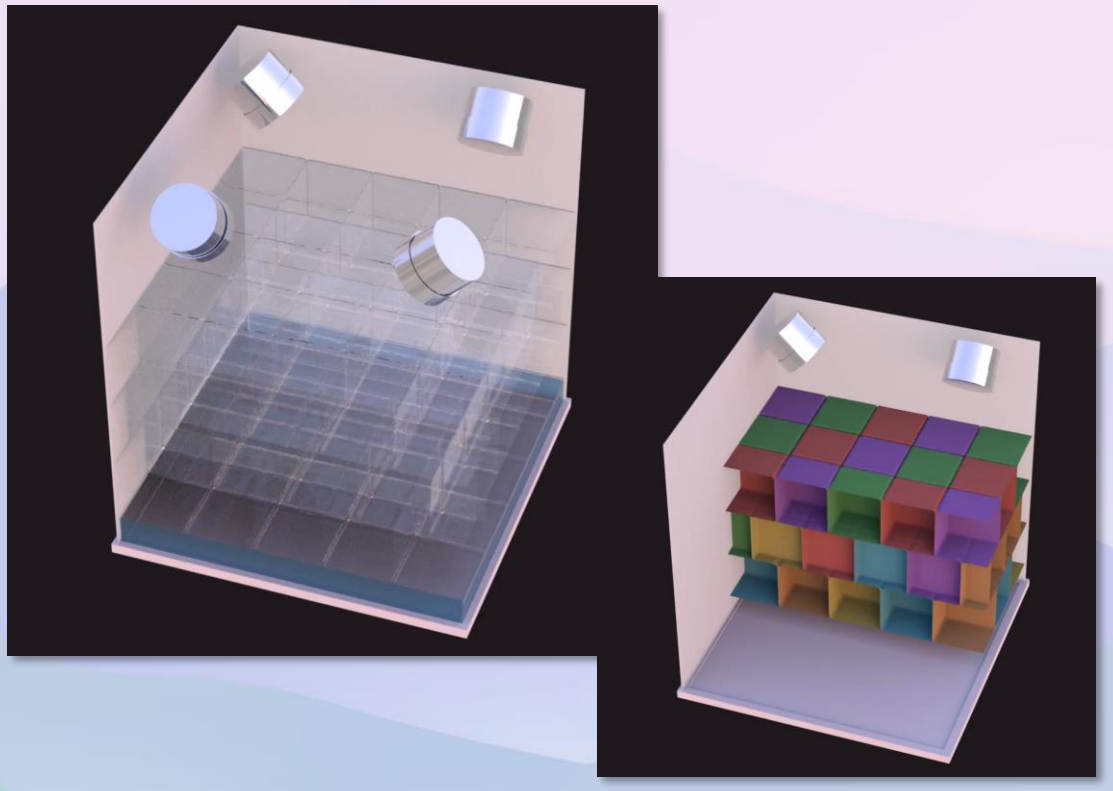
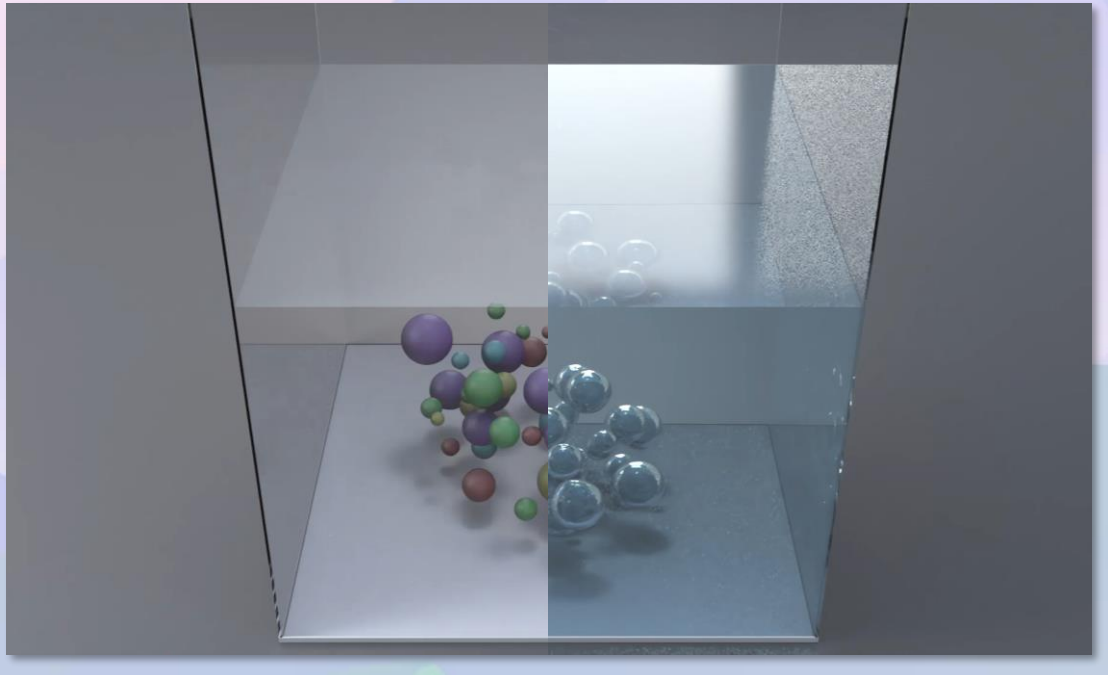
Results: Foaming Flows

Jet on Bubbles

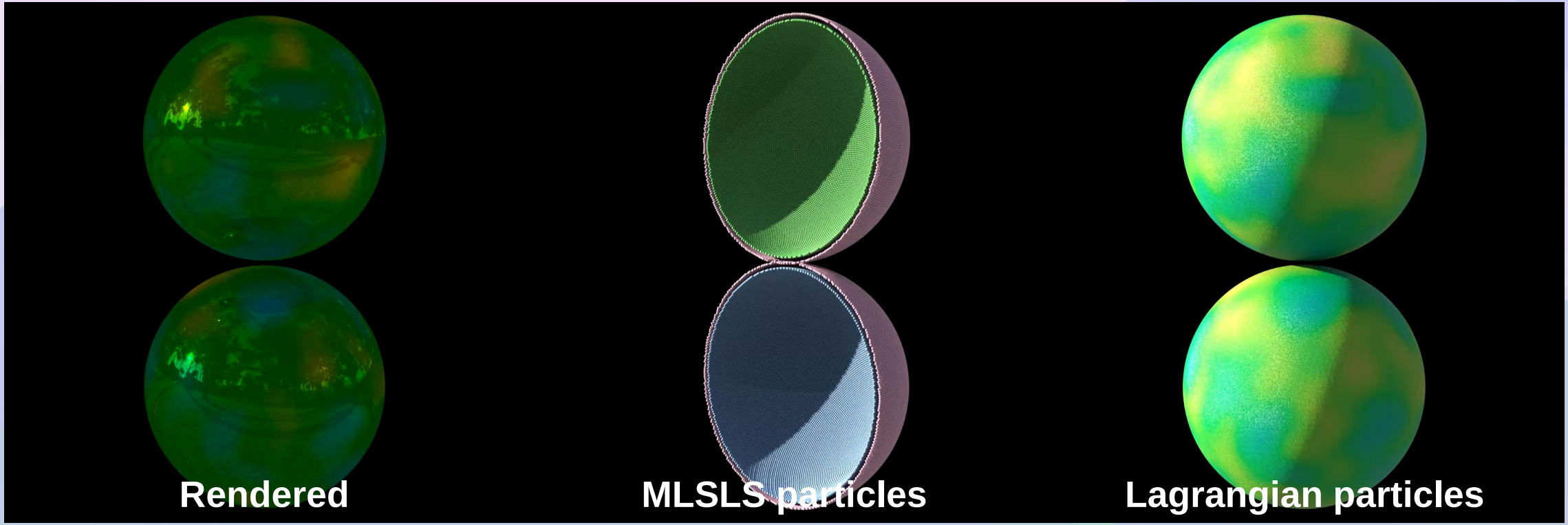


Rising Bubbles

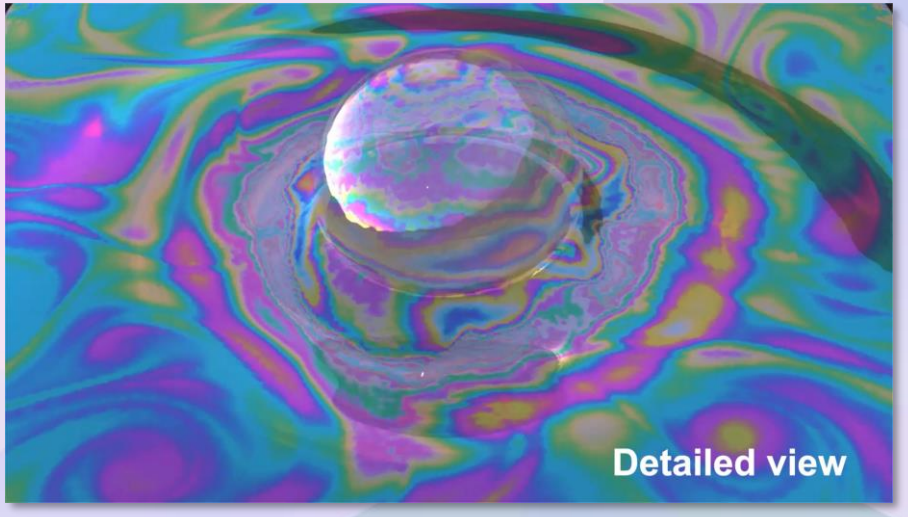


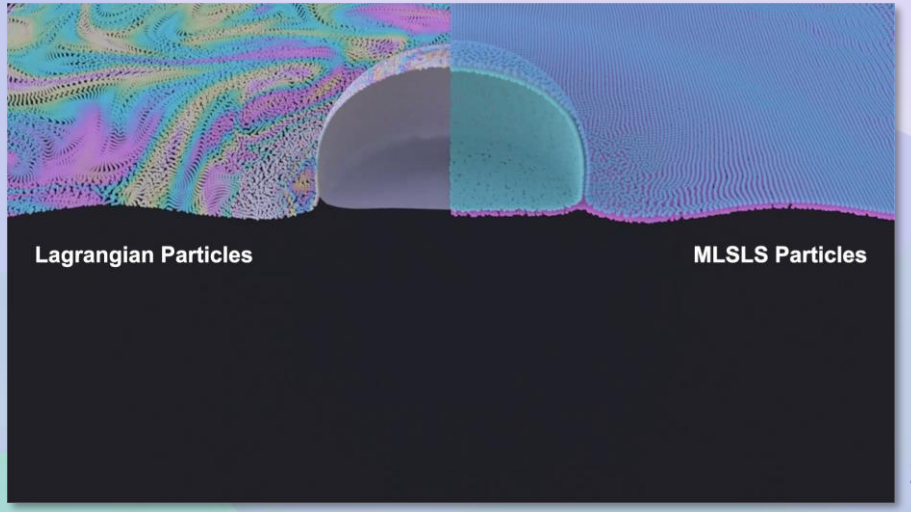
Results: Bubbles

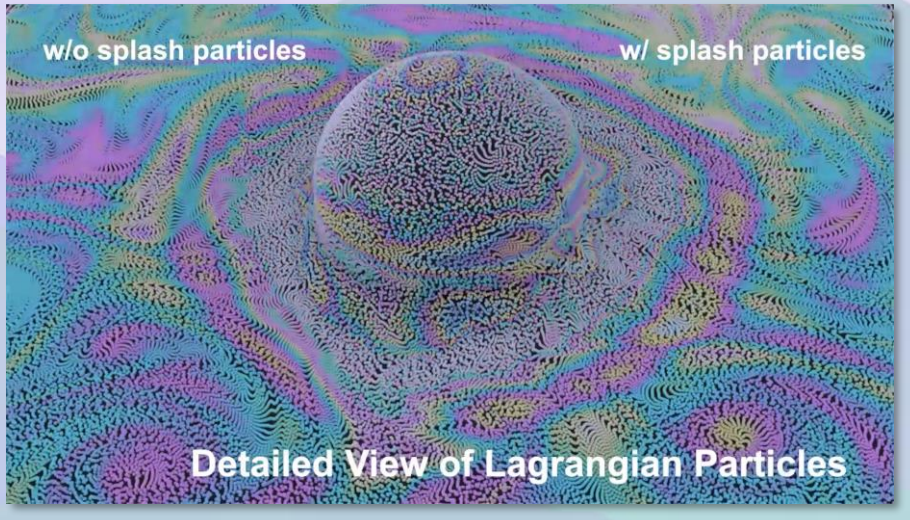
Double bubble



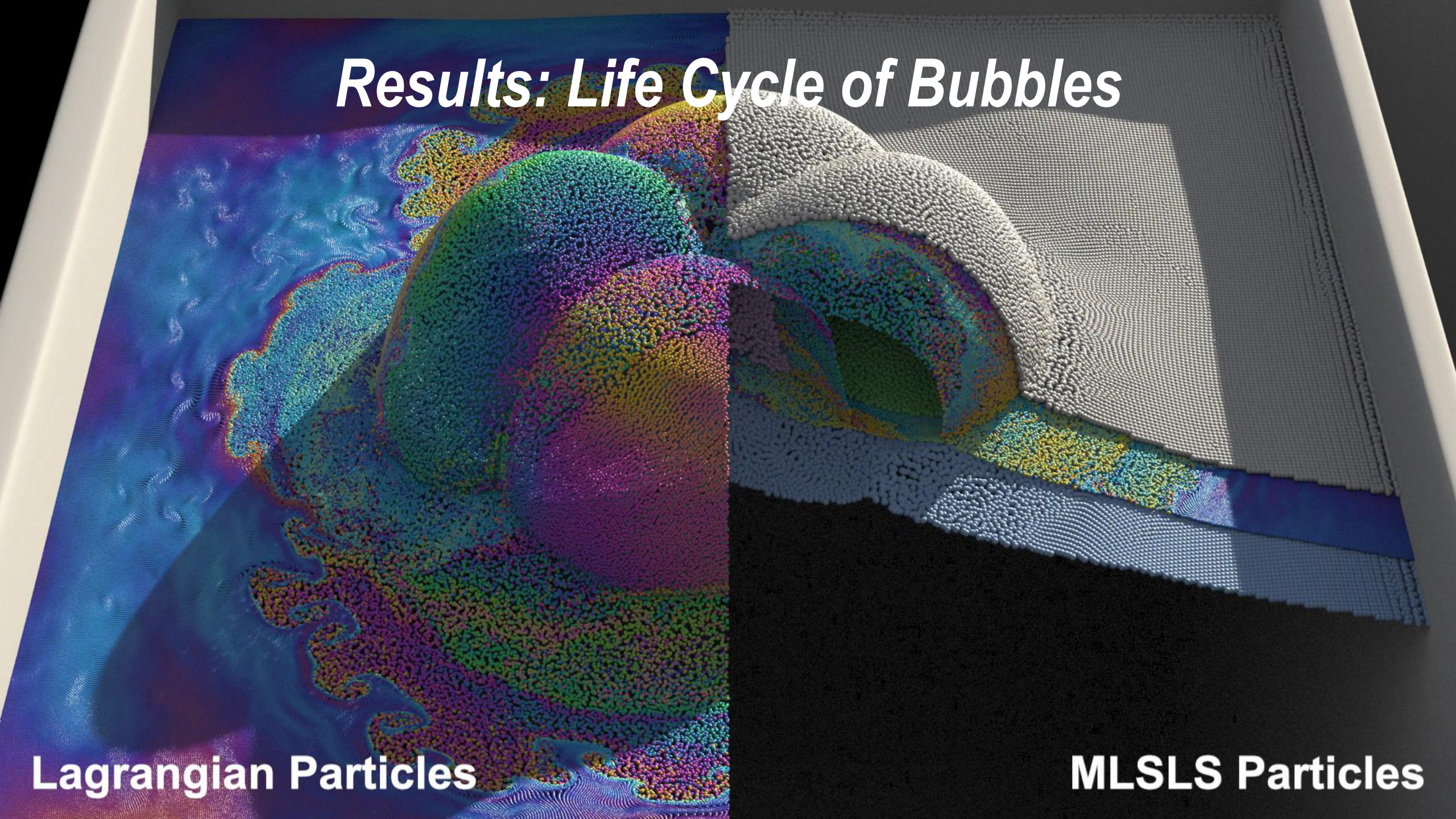
Results: Life Cycle of a Bubble



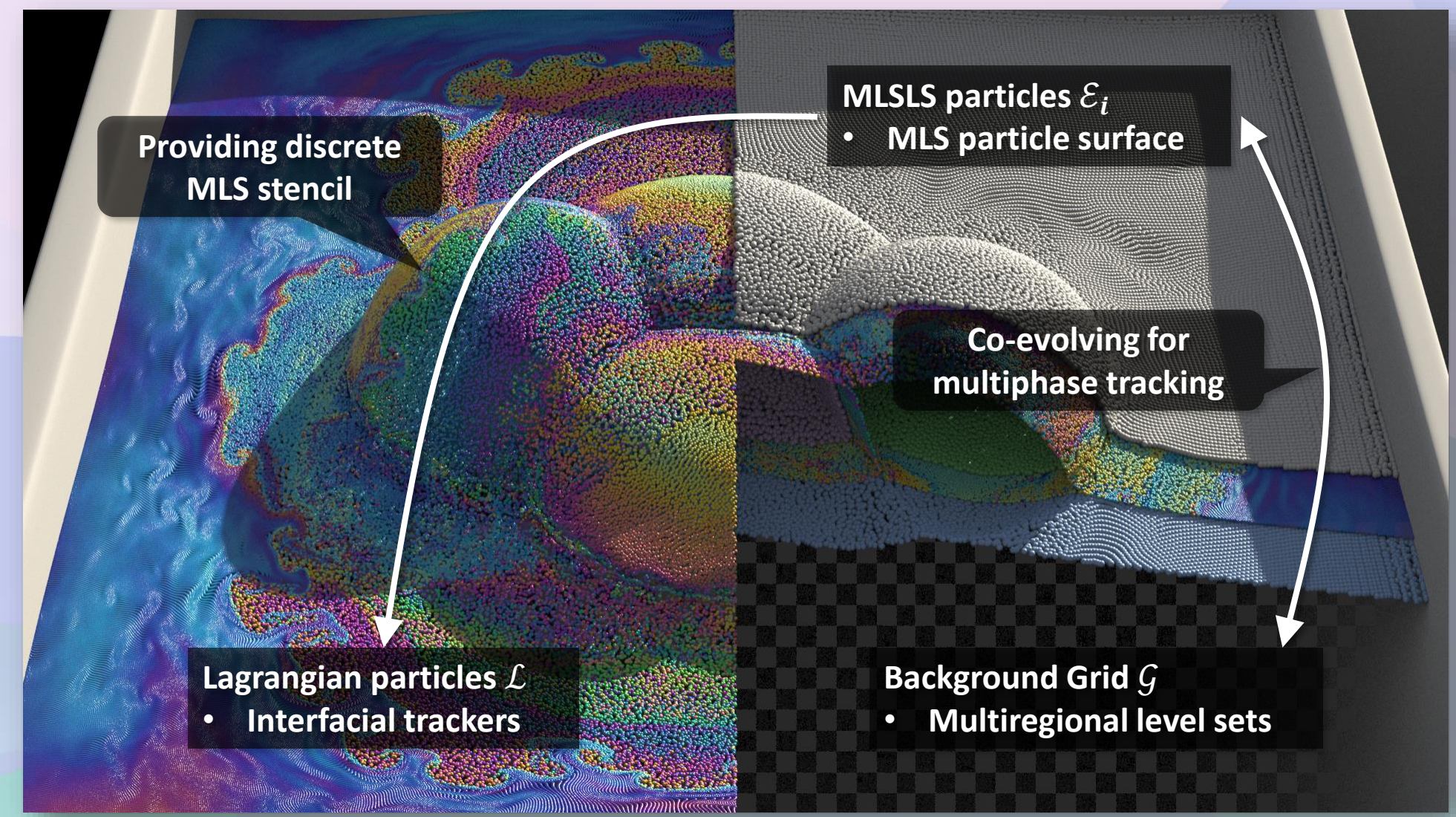




Results: Life Cycle of Bubbles

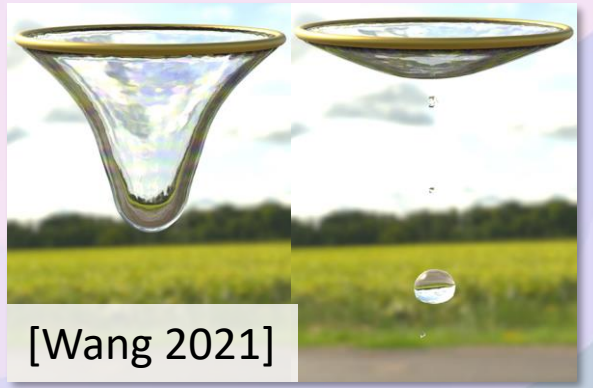


Summary


Providing discrete MLS stencil
MLSLS particles $\mathcal{E}_i$
• MLS particle surface
Co-evolving for multiphase tracking
Lagrangian particles $\mathcal{L}$
• Interfacial trackers
Background Grid $\mathcal{G}$
• Multiregional level sets

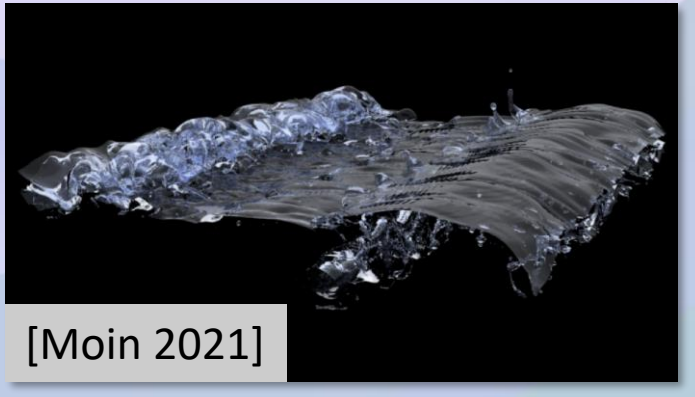
Future work

- Coupled codimensional phenomena
  - Smooth film-volume transition
  - e.g. fluid along the bubble
  - Coupled velocity field
  - e.g. blowing bubble

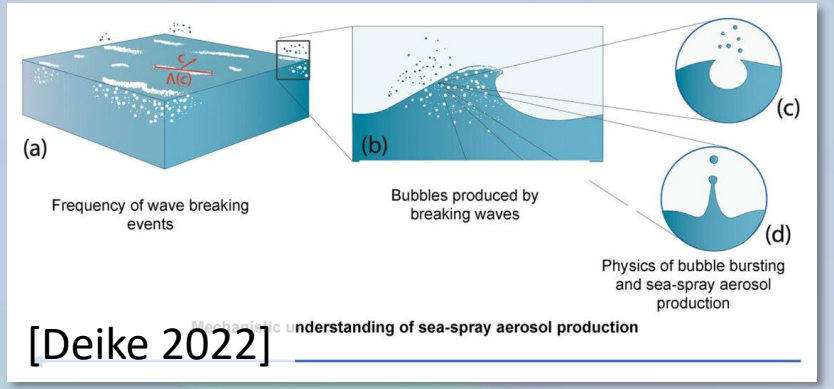


[Wang 2021]

- Bubbles across drastically different scale
  - Cross-scale bubbles
  - e.g. breaking waves



[Moin 2021]



[Deike 2022]

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